

SOLUTION OF MIXED CONVECTION HEAT TRANSFER FROM ISOTHERMAL IN-LINE FINS

Amir Khalilollahi*

The School of Engineering and Engineering Technology
The Pennsylvania State University at Erie, The Behrend College
Erie, PA 16563

SUMMARY

Transient and steady state combined natural and forced convective flows over two in-line finite thickness fins (louvers) in a vertical channel are numerically solved using two methods. The first method of solution is based on the "Simple Arbitrary Lagrangian Eulerian" (SALE) technique which incorporates mainly two computational phases: (1) a Lagrangian phase in which the velocity field is updated by the effects of all forces, and (b) an Eulerian phase that executes all advective fluxes of mass, momentum and energy. The second method of solution uses the finite element code entitled FIDAP. In the first part of this study, comparison of the results by FIDAP, SALE and available experimental work were done and discussed for steady state forced convection over louvered fins. Good agreements were deduced between the three sets of results especially for the flow over a single fin. In the second part of the study and in the absence of experimental literature, the numerical predictions were extended to the transient transports and to the opposing flow where pressure drop is reversed. Results are presented and discussed for heat transfer and pressure drop in assisting and opposing mixed convection flows.

INTRODUCTION

Louver arrays are used to enhance the performance of compact heat exchangers. If the orientation of the exchanger is vertical and the flow rates are low, the buoyancy forces would effect the heat transfer and pressure drag characteristics of the fins. Mixed convection near rectangular fins with finite thickness has been studied by Kurosaki et al [1], Sparrow et al. [2], and Suzuki et al. [3]. Reference [1] has provided experimental data for a single fin, two collinear fins, two parallel fins and a staggered array of fins. Suzuki et al. [3] presented finite difference solutions for an array of very thin fins in assisting (upward) flow and discussed heat transfer characteristics of arrays. Transient mixed convection over a single fin was studied by Khalilollahi and Joshi [4] where temperature overshoots and enhanced heat transfer rates were observed for higher Grashof numbers. The transient and steady state assisting flow over two in-line was numerically investigated in Ref. [5] and steady state results were compared with some experimental data reported in Ref. [1].

*Assistant Professor of Mechanical Engineering

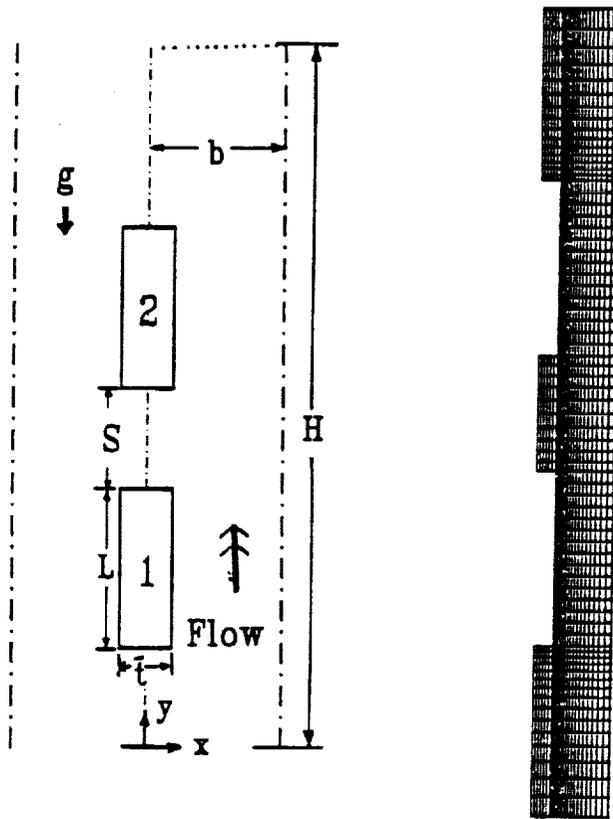


Figure 1. Array geometry and computational domain ($L=.022$ m, $S=.015$ m, $H=.105$ m, $b=.01$ m, $t=.0022$ m)

The present study is intended to (1) enhance the confidence in the solution by the finite difference FORTRAN code, SALE (Simple Arbitrary Lagrangian Eulerian) through comparisons with the solution by the finite element package, FIDAP, and (2) to investigate the opposing convective flow where the pressure difference between the top and bottom sections reverses causing pressure field to oppose the buoyancy force.

NUMERICAL PREDICTIONS

Finite Difference Scheme

Figure 1 shows the model geometry, flow domain and computational grid to the left of x-axis. This domain is used in both SALE and FIDAP solutions. The flow field is governed by the conservation equations in dimensionless form,

$$\partial U/\partial X + \partial V/\partial Y = 0 \quad (1)$$

$$\partial U/\partial \tau + U\partial U/\partial X + V\partial U/\partial Y = -\partial P/\partial X + \partial \Pi_{xx}/\partial x + \partial \Pi_{xy}/\partial Y \quad (2)$$

$$\partial V/\partial \tau + U\partial V/\partial X + V\partial V/\partial Y = -\partial P/\partial Y + \partial \Pi_{yy}/\partial Y + \partial \Pi_{xy}/\partial X + Gr\theta \quad (3)$$

$$\partial \theta/\partial \tau + U\partial \theta/\partial X + V\partial \theta/\partial Y = [\partial^2 \theta/\partial X^2 + \partial^2 \theta/\partial Y^2]/Pr \quad (4)$$

where

$$\Pi_{xx} = 2\partial U/\partial X, \quad \Pi_{yy} = 2\partial V/\partial Y, \quad \Pi_{xy} = \partial U/\partial Y + \partial V/\partial X,$$

$$Gr = g\beta(T_H - T_C)L^3/\nu^2, \quad Pr = \mu C_p/k, \quad Nu = qL/\Delta T k, \quad \theta = (T - T_C)/(T_H - T_C),$$

$$X = x/L, \quad Y = y/L, \quad U = uL/\nu, \quad V = vL/\nu, \quad \tau = t\nu/L^2, \quad P = pL^2/\rho\nu^2$$

The boundary and initial conditions are

$$\tau = 0, \quad U = V = \theta = 0, \quad (\text{initial conditions})$$

$$\tau > 0,$$

$$\partial V/\partial X = U = \partial \theta/\partial X = 0 \quad (\text{at } X = 0, b/L)$$

$$U = V = 0, \quad \theta = 1 \quad (\text{on fin surfaces})$$

$$[P]_{Y=0} - [P]_{Y-H} = \Delta P \quad (\text{pressure drop})$$

The axes of symmetry are at $X=0$ and $X=b/L$, as shown in Fig. 1. The above equations were solved numerically by a Fortran code which incorporates Simple Arbitrary Lagrangian Eulerian

finite difference scheme. This technique is described by Amsden et al. [6]. SALE procedure includes a Lagrangian explicit method where computational cells move with the flow, and an Eulerian phase in which the cells are returned to the original position. This phase estimates the effects of advective fluxes of mass, momentum and energy on the flow parameters.

Finite Element Scheme

The finite element solution of conservation equations governing the laminar flow with boundary/initial conditions in this problem was made possible through the available educational version of the FIDAP package [7]. The grid independence was determined by doubling the number of elements until less than 2% difference in maximum velocity at midsection of the lower fin was observed. The same procedure was incorporated in the finite difference scheme, SALE. An acceleration factor of 0.3, the quasi-Newton solver, pressure penalty formulation, and a fixed time increment (.05 sec) were used. For both schemes, similar unequally sized grid was assigned with higher cell density near the heated fin surfaces (Fig.1). The convergence in all cases was relatively fast and smooth.

RESULTS AND DISCUSSION

Some transient problems in natural and mixed convection flows have been previously solved by using SALE procedure with favorable results [8,9]. This study intends to apply this technique and finite element analysis to the phenomenon of heat dissipation of in-line finite thickness louvers. In the absence of empirical data, the predictions of transients and thermal characteristics of fins in adverse pressure fields can be valuable. In addition, Comparison of the two set of predictions (by FIDAP and SALE) can evaluate the reliability and accuracy of these methods when applied to thermal design problems.

Figure 2 indicates the heat transfer steady state performance of a single rectangular fin in assisting flow where positive pressure drop assists the buoyancy force. Reynolds number range is between 30 to 600. This is common for air-cooled compact heat exchangers. The aspect ratio t/L has been shown to have minor effect on the overall heat transfer rates [1]. The results are shown for Nusselt number vs. Grashof number and for a fin with $t/L = .2$ in infinite (air) medium. The correlation for forced convection flow over a flat plate is (shown in Fig. 2)

$$Nu = .644 Pr^{1/3} Re^{1/2} \quad (5)$$

The agreement between the three sets of predictions is good especially in the range where buoyancy is dominant and Re number is low. The onset of disturbed flow at higher flow rates

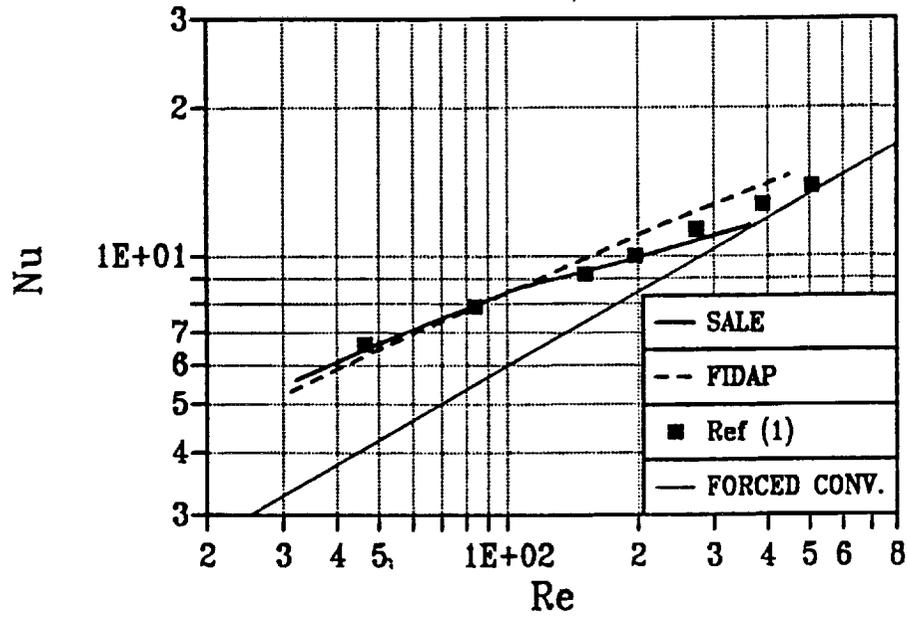


Figure 2. Steady state Nusselt number for single fin in assisting flow

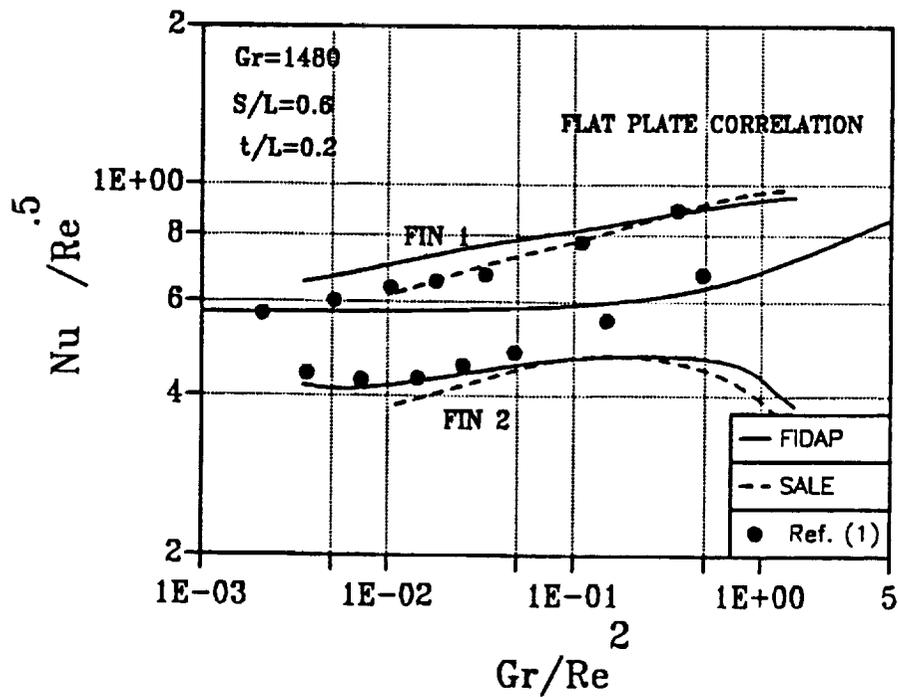


Figure 3. Comparison of steady state Nusselt number for assisting flow

and adjacent to fin corners may be accounted for the small disagreement at $Re > 200$. The experimental values seem to be average of the two numerically predicted sets.

Figure 3 presents the comparison of Nusselt number for assisting steady state flow over two fins (as shown in Fig. 1) which is positioned in an array. The presence of neighboring fins in the array creates the symmetry lines, $Y=0$ and $Y=b/L$. The higher values of Gr/Re^2 represents the dominance of natural convection while lower range of Gr/Re^2 indicates the forced convection regime. The experimental data from Ref. [1] is for in-line fins in infinite medium. This explains the difference between the numerical and experimental values for the downstream fin (#2). However, predictions of FIDAP and SALE are in favorable agreement especially in buoyancy dominant regime. For reference, the relation for mixed convection heat transfer for a flat plate is shown in Fig. 3 [10].

The estimation of pressure drop is an important consideration in design of heat exchangers. In the absence of experimental data for fins of present study, numerical predictions by SALE and FIDAP are presented and compared for pressure coefficient and for pure forced ($Gr=0$) and mixed ($Gr=1480$) convections as shown in Fig.4. The agreement is very good especially for lower Reynolds number ($Re < 40$) where convection starts by buoyancy action. The difference is steady but higher (about 10%) for higher Re numbers where forced convection is dominant.

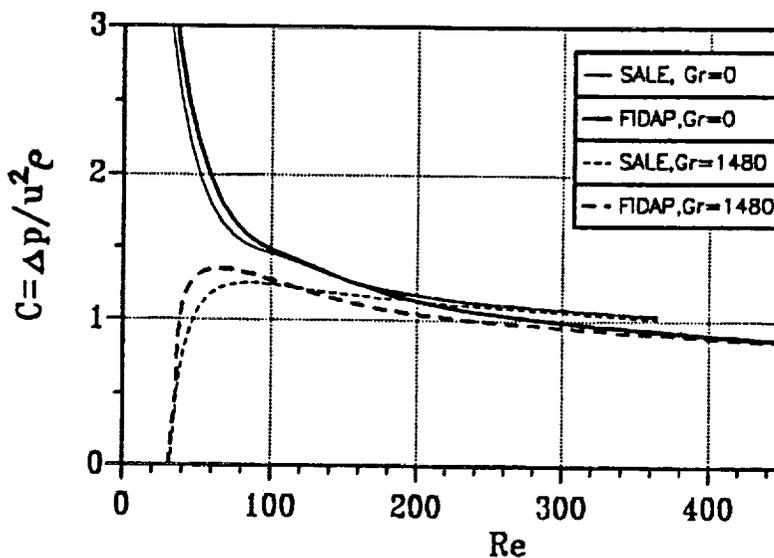


Figure 4. Comparison of pressure coefficient for assisting flow

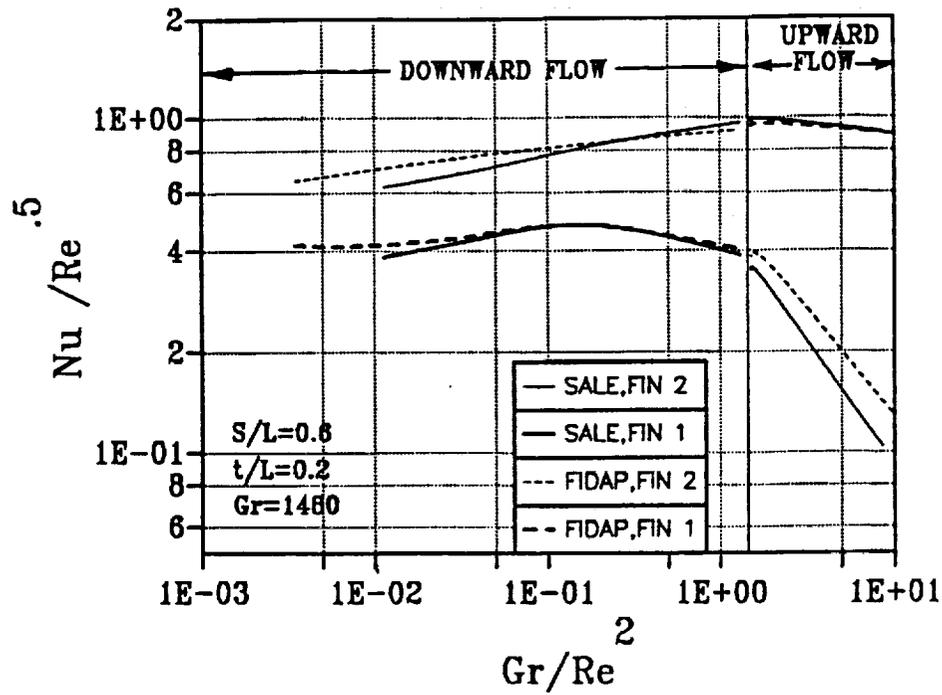


Figure 5. Comparison of steady state Nusselt number for opposing flow

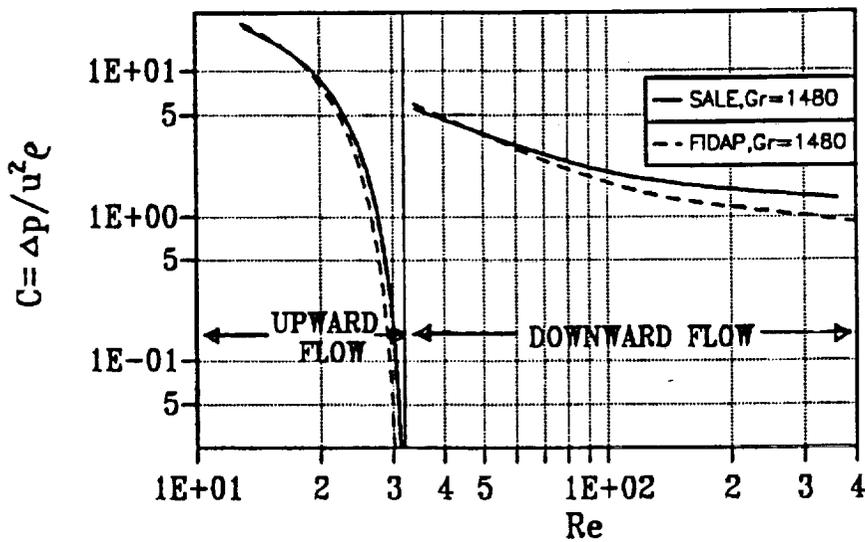


Figure 6. Comparison of pressure coefficient for opposing flow

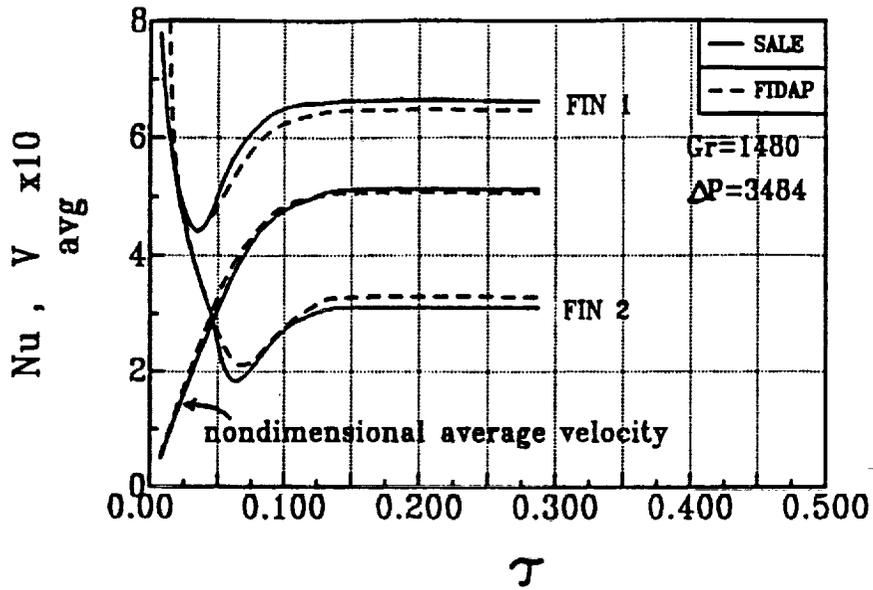


Figure 7. Comparison of transient Nusselt number and average velocity for assisting flow

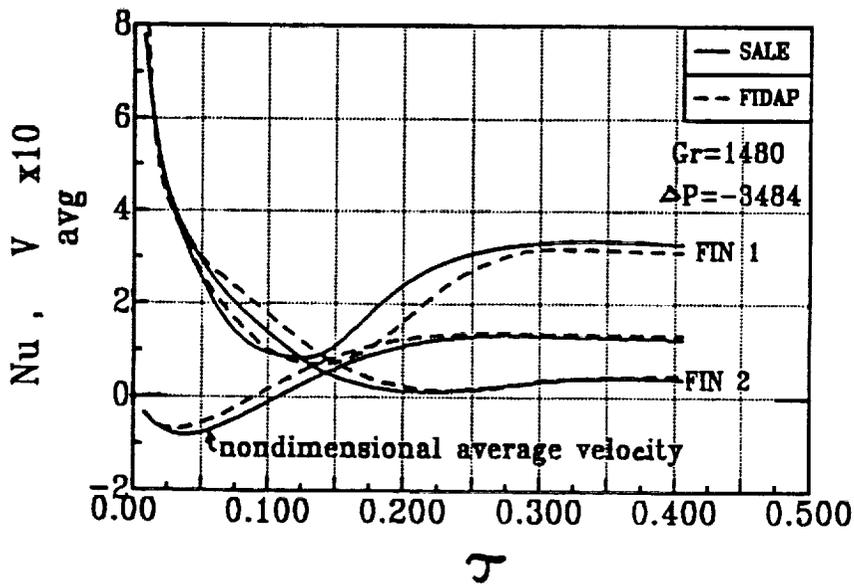


Figure 8. Comparison of transient Nusselt number and average velocity for opposing flow

The term "opposing flow" is given to the case where the pressure drop is reversed. That means the higher pressure at the top section presses the flow downward while opposing the forces of buoyancy. This case presents an interesting discontinuity trend for the flow and heat transfer. Upward free convection begins with $\Delta P=0$ at about $Gr/Re^2=1.3$ ($Re=34$), causing higher heat transfer rates for the downstream fin (#1). Flow stays upward but loses intensity when the magnitude of pressure drop (which is negative) increases. Diminishing flow corresponds to the upper limit of $Gr/Re^2=10$ shown in Fig.5. Increasing the magnitude of ΔP will change the direction of the flow downward not gradually but stepwise where a sudden shift is observed for the Nu number of both fins. Thereafter the flow starts from the upper limit, $Gr/Re^2=1.3$, in downward mixed convection region and moves to lower Gr/Re^2 region approaching pure forced convection. The differences between SALE and FIDAP predictions are more noticeable for the second fin in the upward flow region and for both fins in the forced convection dominant region.

Similar trend is observed for pressure coefficient in opposing flow (Fig. 6). Lower magnitude of ΔP , ($|\Delta P| < 4500$) is not sufficient to push the flow downward in the region where $Re < 33$.

With increasing $|\Delta P|$, The flow vanishes ($Re=0$) but then suddenly changes direction to a downward flow with Re of about 33. Increasing the magnitude of the pressure difference increases Re where differences between FIDAP and SALE results become more noticeable. Overall the agreement seems to be favorable.

Transient heat transfer and average vertical component of velocity for $Gr=1480$ and $\Delta P=\pm 3484$ are shown in Figures 7 and 8, where predictions by SALE and FIDAP are compared. Figure 7 shows the transient Nu vs. dimensionless time, τ , and for assisting flow ($\Delta P=+3484$). The solutions show minima at early times, then reaching steady state values quickly especially for the downstream fin. These minima are caused by the onset of convection and after the early conduction heat transfer lowers. The agreement seems to be fairly good. The underestimation of Nu by FIDAP for fin #1 is opposite to the trend seen for the fin #2. Very good agreement is observed for the average velocity ($V=vL/v$).

Figure 8 represents the same trends observed in Fig. 7. The flow is an opposing type since ΔP is negative. Initially the flow is negative or downward, but eventually it reaches steady upward flow since buoyancy effects are more dominant. At early times ($.05 < \tau < .15$) under downward pressure force, the flow is downward before the strengthening of buoyancy effects. This region presents higher Nu values for fin #2, since it is the upstream fin. Later the flow direction changes while approaching steady state with fin #1 becoming the upstream fin. The discrepancy between FIDAP and SALE is moderately significant for the first fin and at about $\tau=0.2$. The agreement for the average velocity is fine except at intermediate times ($.05 < \tau < .25$) when FIDAP overestimates the predictions by SALE.

CONCLUSION

Transient and steady state heat transfer characteristics of mixed convection were analyzed for assisting and opposing flows over two in-line vertical isothermal fins. Steady state assisting flow was predicted by three means, namely FIDAP and SALE that are numerical schemes, and

by available experimental data in the literature. The trends in pressure coefficient were presented for above cases. A discontinuous trend in Nu number, flow rate, and pressure coefficient was observed for negative ΔP values (opposing flow). Overall predictions by the two schemes, FIDAP and SALE, compared favorably. Future work is planned to study further this discontinuity and to conduct experiments on verification of numerical results especially for opposing flows.

REFERENCES

1. Kurosaki, Y., Kashiwagi, T., Kobayashi, H., Uzuhashi, H., and Tang, S., "Experimental Study on Heat Transfer from Parallel Louvered Fins by Laser Holographic Interferometry", *Experimental thermal and Fluid Science*, 1, 59-67, 1988.
2. Sparrow, E. M., Baliga, B. R., and Patankar, S. V., "Heat Transfer and Fluid Flow Analysis of Interrupted Wall Channels, with Application to heat exchangers", *J. Heat Transfer*, 99, #1, 4-11, 1977.
3. Suzuki, K., Hirai, E., Sato, T., and Kieda, S., "Numerical Study of Heat Transfer System with Staggered Array of Vertical flat Plates used at Low Reynolds Number", *Heat Transfer* 1982, 3, 483-488, Hemisphere, N. Y., 1982.
4. Khalilollahi A., and Joshi, H., "Transient and Steady State Solutions for Laminar Flow over a Finite Thickness Fin in a Parallel Plate Channel", *Proc. ASME N. H. T. Conf.*, Houston, July 24-27, 1, 439-444, 1988.
5. Khalilollahi, A., "Transient Combined Free and Forced Convection over Two In-Line Isothermal Rectangular Fins", *Computers and Experiments in Fluid Flow*, Springer-Verlag, 429-438, 1989.
6. Amsden, A. A., Ruppel, H.M., and Hirt, C.W., "SALE: A Simplified ALE Computer Program for Fluid Flows at all Speeds", Report No. LA-8095, Los Alamos Scientific Lab., NM., 1980.
7. FIDAP Manuals, Fluid Dynamics International, Evanston, IL., 1990.
8. Khalilollahi A, and Sammakia, B., "Unsteady Natural Convection Generated by a Heated Surface within an Enclosure," *Numerical Heat Transfer*, 715-730, 1986.
9. Khalilollahi A., and Sammakia B., "The Thermal Capacity Effect upon Transient Natural Convection in a Rectangular cavity", *ASME J. Electronic Packaging*, 112, 357-366, 1990.
10. Churchill, S.W., "A Comprehensive Correlating Equation for Laminar, Assisting, Forced, and Free Convection", *AICHE J.*, 23, #1, 10-16, 1977.